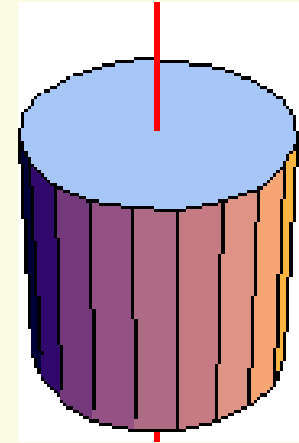


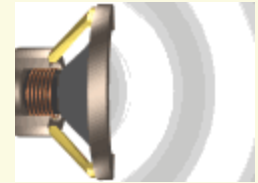
## Aula 7: Movimento do corpo rígido

1. Translação e Rotação
2. Momento angular
3. Segunda lei de Newton para rotação
4. Momento de inércia de corpos homogéneos
5. Regra de Routh



# Movimento do corpo rígido: animação

## Simulação: rotação do corpo rígido

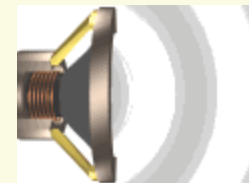


### 1. Translação e Rotação

$$v = v_0 \pm at \Rightarrow \frac{v}{r} = \frac{v_0}{r} \pm \frac{a}{r}t \Rightarrow \omega = \omega_0 \pm \varepsilon t \quad s = v_0 t \pm \frac{1}{2}at^2 \Rightarrow \frac{s}{r} = \frac{v_0}{r}t \pm \frac{1}{2}\frac{a}{r}t^2 \Rightarrow \theta = \omega t \pm \frac{1}{2}\varepsilon t^2$$

Translação	Rotação
$m$	$I = \sum mr^2$
$F$	$\mu = Fr$
$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$
$p = mv$	$L = I\omega$

Translação	Rotação
$ma = F$	$I\varepsilon = \mu$
$mv_f - mv_i = F\Delta t$	$I\omega_f - I\omega_i = \mu\Delta t$
$W = Fs$	$W = \mu\theta$
$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W$	$\frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W$
$P = Fv$	$P = \mu\omega$

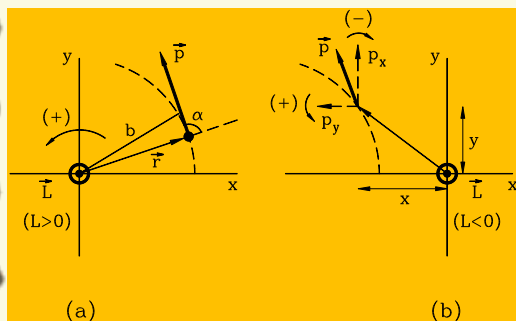


## Simulação: produto vectorial

## Simulação: momento de torção

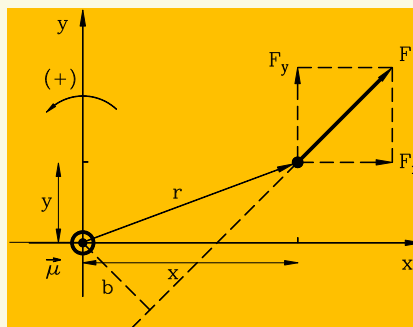
## 2. Momento angular

$$\vec{L} = \vec{r} \times \vec{p}$$



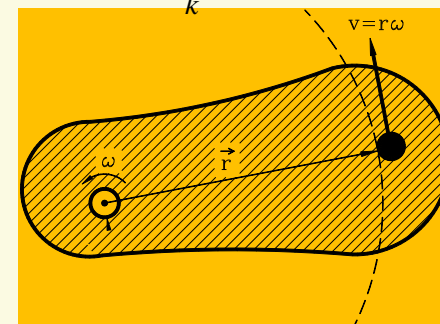
### Momento de torção

$$\vec{\mu} = \vec{r} \times \vec{F}$$

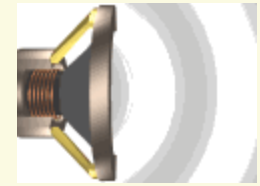


### Momento de inércia

$$I = \sum_k m_k r_k^2$$

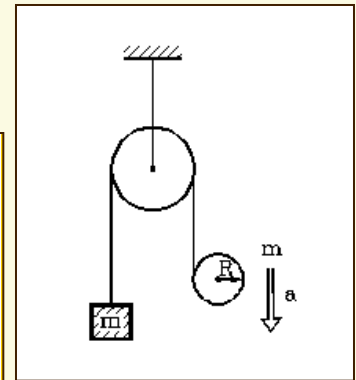
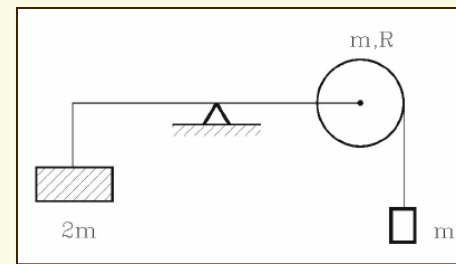
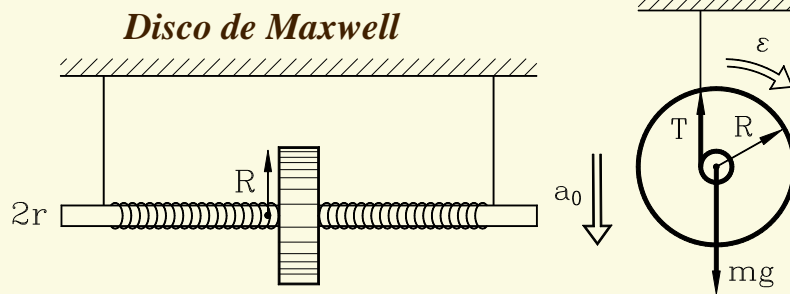


# Simulação: sistema de roldanas



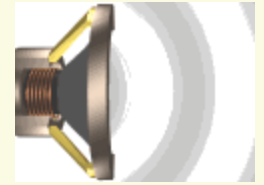
## 3. Segunda lei de Newton para rotação

$$m\vec{a} = \sum_k \vec{F}_k \quad \rightarrow \quad I\alpha = \sum (\pm \mu_{ext})$$

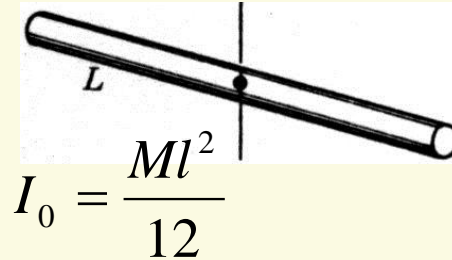
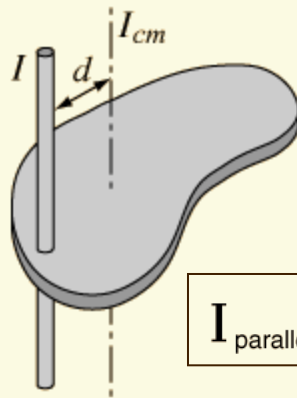


## Teorema de Steiner: animação

## Simulação: momento de inércia

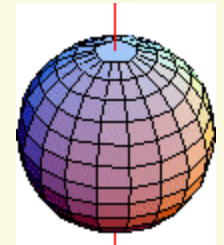


### 4. Momento de inércia de corpos homogéneos

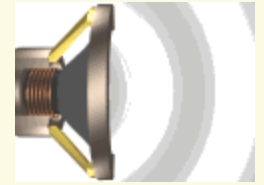


$$I_{\text{parallel axis}} = I_{\text{cm}} + Md^2$$

$$I_0 = \frac{MR^2}{2}$$

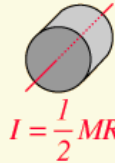


$$I_0 = \frac{2MR^2}{5}$$



## 5. Regra de Routh

Solid cylinder or disc, symmetry axis



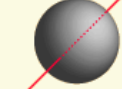
$$I = \frac{1}{2}MR^2$$

Hoop about symmetry axis



$$I = MR^2$$

Solid sphere



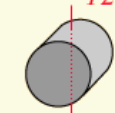
$$I = \frac{2}{5}MR^2$$

Rod about center



$$I = \frac{1}{12}ML^2$$

$$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$$



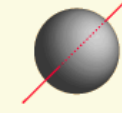
Solid cylinder, central diameter

$$I = \frac{1}{2}MR^2$$



Hoop about diameter

$$I = \frac{2}{3}MR^2$$



Thin spherical shell

$$I = \frac{1}{3}ML^2$$



Rod about end

$$I_{CM} = M \left( \frac{a^2 + b^2}{n} \right)$$

$n = 3$  para corpos *paralelepípedicos*

$n = 4$  para corpos *cilíndricos*

$n = 5$  para corpos *esféricos*