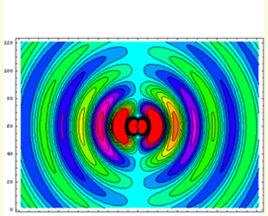


Aula 15: Equações da electrostática

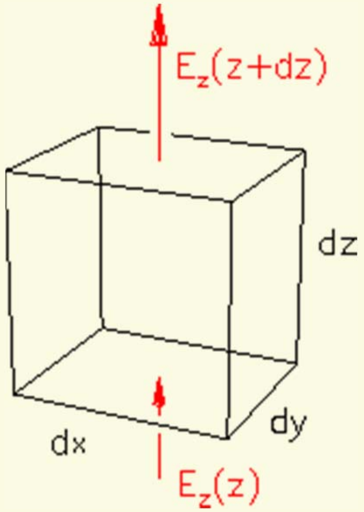
15.1. Forma local da lei de Gauss

15.2. Lei de Gauss para dieléctricos

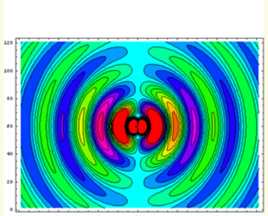
15.3. Forma local da equação de continuidade

15.4. Equações de Poisson e de Laplace

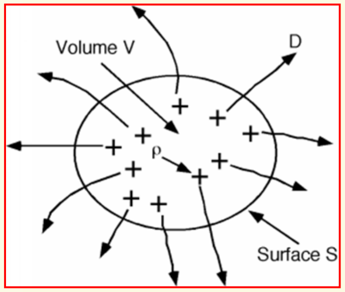
15.5. Forma local da lei de Faraday



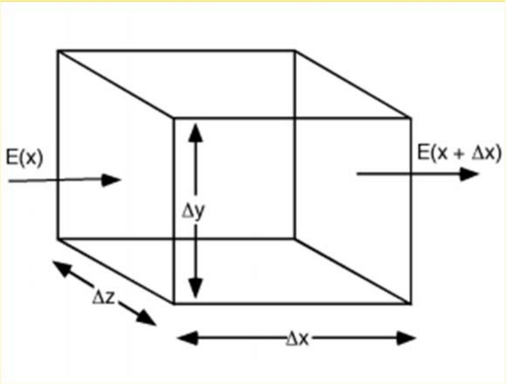
Fluxo eléctrico: animação



15.1. Forma local da lei de Gauss



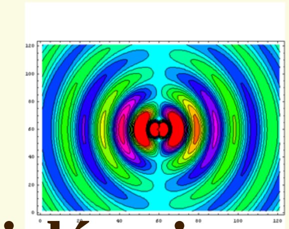
$$\int_S \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \int_V \rho dV$$



$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{1}{\varepsilon_0} \rho$$

Operador nabla: $\nabla = \vec{e}_x \frac{\partial}{\partial x} + \vec{e}_y \frac{\partial}{\partial y} + \vec{e}_z \frac{\partial}{\partial z}$

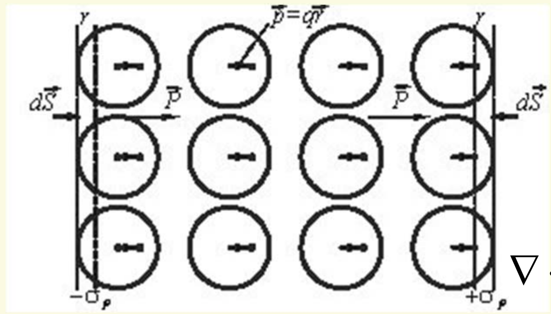
$$\boxed{\nabla \cdot \vec{E} = \frac{1}{\varepsilon_0} \rho}$$



15.2. Lei de Gauss para dieléctricos

Teorema de divergência:

$$\boxed{\int_{V}\left(\nabla\cdot\vec{A}\right)dV=\int_{S}\vec{A}\cdot d\vec{S}}$$



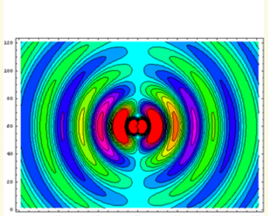
$$\int_{S}\sigma_P dS=\int_{S}\vec{P}\cdot d\vec{S}=\int_{V}\nabla\cdot\vec{P}\,dV$$

$$\int_{S}\sigma_P dS+\int_{V}\rho_P dV=0$$

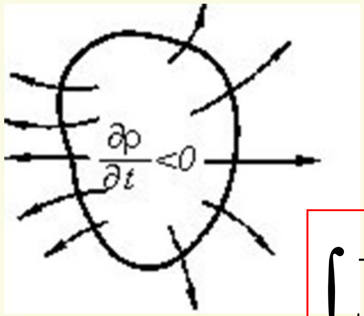
$$\rho_P=-div\,\vec{P}=-\nabla\cdot\vec{P}$$

$$\nabla\cdot\vec{E}=\frac{1}{\varepsilon_0}\left(\rho+\rho_P\right)=\frac{1}{\varepsilon_0}\left(\rho-\nabla\cdot\vec{P}\right)$$

$$\boxed{\nabla\cdot\vec{D}=div\,\vec{D}=\rho}$$



15.3. Forma local da equação de continuidade

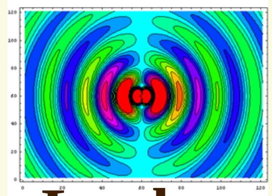


$$\int_S \vec{j}\cdot d\vec{S} = \int_V \left(\nabla\cdot\vec{j}\right) dV$$

$$\boxed{\int_S \vec{j}\cdot d\vec{S} = -\frac{\partial}{\partial t}\int_V \rho\, dV = -\int_V \frac{\partial\rho}{\partial t}\, dV}$$

Equação de continuidade:

$$\nabla\cdot\vec{j} + \frac{\partial\rho}{\partial t} = 0$$



15.4. Equações de Poisson e de Laplace

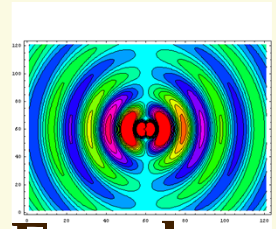
$$\boxed{\nabla\cdot\vec{E}=\frac{1}{\varepsilon_0}\rho}$$

$$\nabla\cdot\left(-\nabla V\right)=-\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}\right)=\frac{1}{\varepsilon_0}\rho$$

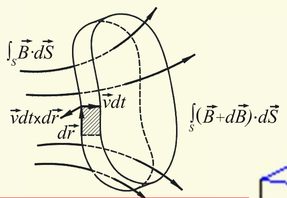
$$\boxed{\vec{E}=-grad\ V=-\nabla V}$$

Equação de Poisson: $$\nabla^2 V=-\frac{1}{\varepsilon_0}\rho$$

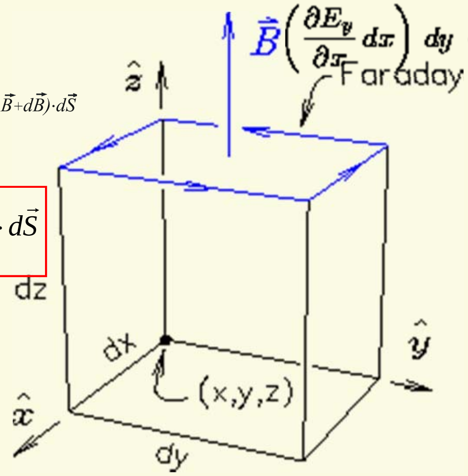
Equação de Laplace: $\nabla^2 V=0$ ou $\dfrac{\partial^2 V}{\partial x^2}+\dfrac{\partial^2 V}{\partial y^2}+\dfrac{\partial^2 V}{\partial z^2}=0$



15.5. Forma local da lei de Faraday



$$\boxed{\int_\Gamma \vec{E}\cdot d\vec{r} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S}}$$



$$\left(\frac{\partial E_y}{\partial x}dx\right)dy - \left(\frac{\partial E_x}{\partial y}dy\right)dx = -\left(\frac{\partial B_z}{\partial t}\right)dx\,dy \Rightarrow \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = -\left(\frac{\partial B_z}{\partial t}\right)$$

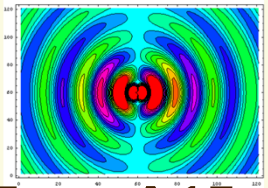
Operador rotacional:

$$\nabla\times\vec{E} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \vec{e}_x\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \vec{e}_y\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \vec{e}_z\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

Lei de Faraday:

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{dt}$$



Teste A15

http://eo.tagus.ist.utl.pt/